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# SIERPINSKIZED KOCH-LIKE SIDED MULTIFRACTAL DIPOLE ANTENNA

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**Abstract**—Koch-like fractal curve and Sierpinski Gasket are syncretized into a novel Sierpinskized Koch-like sided bow-tie (SKLB) multifractal in superior-inferior way. A $K_4S_4$ SKLB multifractal dipole fed by a linearly tapered microstrip Balun is designed, simulated, fabricated and measured. The well consistent results from measurement and experiment corroborate validity of design and the multifractal antenna's superiority and advantages over its monofractal counterparts in impedance, bandwidth, directivity, efficiency, and dimension. Six good matched bands ($S_{11} \leq -10\,\text{dB}$) with moderate gain (2.12 dBi–9.55 dBi) and high efficiency (87%–97%) are obtained within band 1.5 GHz–14.5 GHz, of which $f_1 = 1.92\,\text{GHz}$, $f_2 = 3.94\,\text{GHz}$, and $f_3 = 5.09\,\text{GHz}$ are generally useful. The multibands are all almost omnidirectional or quasi-omnidirectional in $H$-plane ($Phi = 0°$, $XOZ$) and doughnut-shaped or dented doughnut-shaped in $E$-plane ($Phi = 90°$, $YOZ$). So it is an attractive candidate for applications like PCS, IMT2000, UMTS, WLAN, WiFi, WiMAX and other fixed or mobile wireless multiband communication systems.

## 1. INTRODUCTION

RACTAL antenna has drawn much attention since its introduction in 1995 by Nathan Cohen [1, 2]. It is a combination of antenna technology and fractal geometry [3] and has shown many particular attributes during extensive researches and applications as concluded in [4, 5]. Fractal antenna usually comprises monofractal, which has only one fractal scale ratio, so it essentially has multiband of single frequency ratio [6–8] though with variable scale ratios [9, 10].

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Naturally, we conceive the idea of fabricating multifractal antenna from several monofractals with different scale ratios, so that we can design arbitrary multiband antennas more easily. Those component monofractals are usually coalesced in superior-inferior or main-minor way. So, multifractal antenna often behaviors like the main monofractal as well as resembles the minor one in impedance property and radiation patterns. It reserves the component monofractals' merits and surmounts their demerits simultaneously. In conclusion, multifractal antenna is closely relevant to the monofractals' properties and their combinative way [11]. Multifractal antenna has brought forth significant advantages over monofractal antenna, such as multiband with multiple frequency ratios, further dimension shrinkage and directivity enhancement [9]. Unfortunately, multifractal hasn't been substantially explored for antenna design. Therefore, it is a promising topic of fractal antennas and deserves to be ulteriorly investigated and developed.

Koch-like curve [12] and Sierpinski Gasket are syncretized in main-minor way, comprising so called Sierpinskized Koch-like sided bow-tie (SKLB) multifractal. A SKLB multfractal dipole fed by a linearly tapered microstrip Balun was designed, optimized, fabricated and measured. Good agreement is acquired between simulation and measurement. Like the KSSG counterpart [11], SKLB multifractal dipole also presents conspicuous multifractal properties in impedance, directivity, efficiency, and dimension. Particularly, it shows more remarkable consistency and conspicuous array effect in radiation patterns.

## 2. SIERPINSKIZED KOCH-LIKE SIDED BOW-TIE (SKLB)

According to the viewpoints concluded in [11], multifractal usually consists of several monofractals and behaves intimately with the combinative way. Koch-like curve [12] and Sierpinski Gasket are coalesced in superior-inferior way, which is just opposite to that of KSSG multifractal in [11]. For intuitive comprehension of the proposed multifractal, IFS [13, 14] is not adopted for its description. An isosceles triangle (bow-tie) is fractalized with $K_i$-iterated Koch-like curve on all the sides then a $S_j$-iterated Sierpinski Gasket with $K_n$-iterated ($n = 1, 2 \ldots i$) Koch-like sides is hollowed out from the Koch-like fractalized bow-tie, so we obtain $K_iS_j$ Sierpinskized Koch-like sided multifractal bow-tie, called $K_iS_j$ ($K_i$-Koch-like, $S_j$-Sierpinski Gasket) SKLB for simplicity, as shown in Fig. 1–Fig. 2. The SKLB multifractal is fully parameterized modeled with Ansoft HFSS$^{\text{TM}}$ v.13. The parameters' symbols and meanings are as follows: $\theta_k$, $\theta_s$ is base angle of the initial isosceles triangle of Koch-like bow-tie and the hollowed initial isosceles triangle of Sierpinski Gasket respectively, $\varphi_k$, $\varphi_s$ is base angle of each iterative isosceles-triangular notch of Koch-like bow-tie and Sierpinski Gasket separately; $b_i$ is rectilinear base side length of un-hollowed vertexal isosceles triangle of the Koch-like bow-tie; $d_{Ki}$ are rectilinear base side length of the lateral $K_i$-iterated isosceles triangular notches; $L_{sj}$ are rectilinear lateral side lengths of the hollowed isosceles triangles of $S_j$-iterated Sierpinski Gasket; $D_{sj}$ are rectilinear distance between lateral vertices of the inverted isosceles triangle corresponding to the $S_j$-iterated hollowed isosceles triangles, which are formed with sharp-angled bulges on the Koch-like fractal sides; $\mu$ is height of the Koch-like isosceles triangle bow-tie initiator. All the signs are illustrated, as shown in Fig. 1–Fig. 2. There are some relationships among these arguments:

$$\sigma_{Ki} = \frac{b_{i+1}}{b_i} = \frac{1}{2\cdot(1+\delta_i\cdot\cos\theta_k)} \Leftrightarrow \delta_i = \frac{\left(\frac{1}{2\cdot\sigma_{Ki}}-1\right)}{\cos\theta_k} = \frac{(b_i-2\cdot b_{i+1})}{2\cdot b_{i+1}\cdot\cos\theta_k}$$

$$b_0 = b_n\cdot 2^n\cdot\prod_{i=1}^{n}(1+\delta i\cdot\cos\theta k)(i=0,1,2\ldots n) \tag{1}$$

$$0 < \rho_{Sj} = \frac{L_{Sj}}{D_{Sj}} < 1, \tag{2}$$

where $n$ is the largest iterative number, $\sigma_{Ki}$ the ratio of $b_{i+1}$ to $b_i$ and also fractal scale ratio of contiguous iterations of the Koch-like curve, $\delta_i$ the ratio of base side length of $K_i$-iterated notch to rectilinear length of $K_i$-iterated Koch-like curve, and $\rho_{Sj}$ the ratio of $L_{Sj}$ to $D_{Sj}$

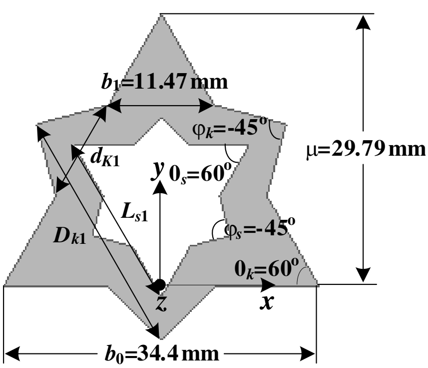



**Figure 1.** $K_1S_1$ SKLB, $b_1 = 11.47\,\text{mm}$.

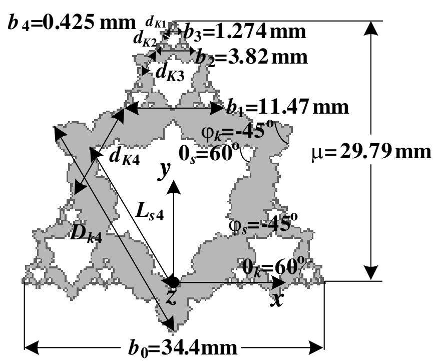



**Figure 2.** $K_4S_4$ SKLB, $b_4 = 0.425\,\text{mm}$.

and also size scale of the hollowed isosceles triangle of $S_j$-iterated Sierpinski Gasket. Intuitively, it depends upon $\rho_{Sj}$ that how much SKLB multifractal behaves like the main monofractal (Koch-like curve) or resembles the minor one (Sierpinski Gasket) in electrical property. Conspicuously, the Koch-like curve is formed with variable fractal scale ratio among each iterative, which is equal to $\chi^{-1} = \frac{\alpha-1}{2\cdot\alpha}$ in its original literature [11]. So, we get the following relationship:

$$\sigma_{Ki} = \frac{1}{2\cdot(1+\delta_i\cdot\cos\theta_k)} = \frac{\alpha-1}{2\cdot\alpha} \Leftrightarrow \delta_i = \frac{1}{(\alpha-1)\cdot\cos\theta_k}, \tag{3}$$

so the fractal scale ratio of Sierpinski Gasket is:

$$\begin{aligned}\sigma_{Sj} &= \rho_{Sj}\cdot\left[1-\frac{1+\delta_i\cdot\cos(\theta_k-\varphi_k)}{2\cdot(1+\delta_i\cdot\cos\theta_k)\cdot\cos\varphi_k}\right]\\ &= \rho_{Sj}\cdot\{1-\sec\varphi_k\cdot[1+\delta_i\cdot\cos(\theta_k-\varphi_k)]\cdot\sigma_{Ki}\}\end{aligned} \tag{4}$$

If $\sigma_{Sj} = \sigma_{Ki}$, from formula (3) and (4), we get:

$$\begin{aligned}\rho_{Sj} &= \frac{\sigma_{Sj}}{1-\sec\varphi_k\cdot[1+\delta_i\cdot\cos(\theta_k-\varphi_k)]\cdot\sigma_{Ki}}\\ &= \frac{1}{2\cdot(1+\delta_i\cdot\cos\theta_k)-\sec\varphi_k\cdot[1+\delta_i\cdot\cos(\theta_k-\varphi_k)]}\end{aligned} \tag{5}$$

Then from formula (1), (2) and (5), we obtain:

$$\begin{aligned}0<\rho_{Sj}<1 &\Rightarrow 2\cdot(1+\delta_i\cdot\cos\theta_k)\cdot\cos\varphi_k-[1+\delta_i\cdot\cos(\theta_k-\varphi_k)]>\cos\varphi_k\\ &\Rightarrow \cos\varphi_k+\delta_i\cdot\cos(\theta_k+\varphi_k)>1\end{aligned} \tag{6}$$

$$d_{i+1} = d_i \Rightarrow \delta_{i+1} = \frac{\sec\theta_k}{2}\cdot\left(1-\frac{\frac{1}{2}}{1+\delta_i\cdot\cos\theta_k}\right) \tag{7}$$

where $\alpha$ is the ratio of base side length of the protrusive notch isosceles triangle to that of the initiator isosceles triangle $b_0$, as illustrated in Fig. 2. Here, $\theta_k = \theta_s = 60°$, $\varphi_k = \varphi_s = -45°$, $b_0 = 34.4\,\text{mm}$, $\mu = 29.79\,\text{mm}$, and $\delta_i = 1\,(i = 1, 2, 3\ldots)$ are chosen for good illustration and more convenience. In addition, a more interesting discovery can be found from the multifractal geometry. The discovery is that fractal scale ratio of Koch-like sided Sierpinski Gasket $\sigma_{Sj}$ is correlated to that of Koch-like curve $\sigma_{Ki}$ with formula (4), and it is not the classic value 0.5 anymore. But for the KSSG counterpart [11], the two ratios are absolutely independent of each other. Apparently, $K_iS_j$ SKLB multifractal is alterable, which possesses great geometric flexibility and performance adjustability. For example, when $\rho_{Sj} = 0$, $\varphi_k = 0°$, and $\rho_{Sj} = 1$, $\delta_i = 0$, $K_iS_j$ SKLB multifractal metamorphoses into $K_i$-iterated Koch-like sided bow-tie $K_iS_0$ and $S_j$-iterated Sierpinski Gasket $K_0S_j$ separately.

## 3. $K_4S_4$ SKLB MULTIFRACTAL DIPOLE ANTENNA

### 3.1. Physical Design of the Multifractal Dipole

$K_4S_4$ SKLB multifractal is chosen as a pragmatic antenna solution for remarkable multifractal impedance property, significant size reduction, more enhanced radiation patterns and geometrical simpleness. We endowed the multifractal dipole with a set of optimum parameters yielded by optimization utilities Genetic Algorithm (GA) [15] and Parametric Sweep of Ansoft HFSS$^{\text{TM}}$ v.13 Optimetrics as: $\theta_k$ =

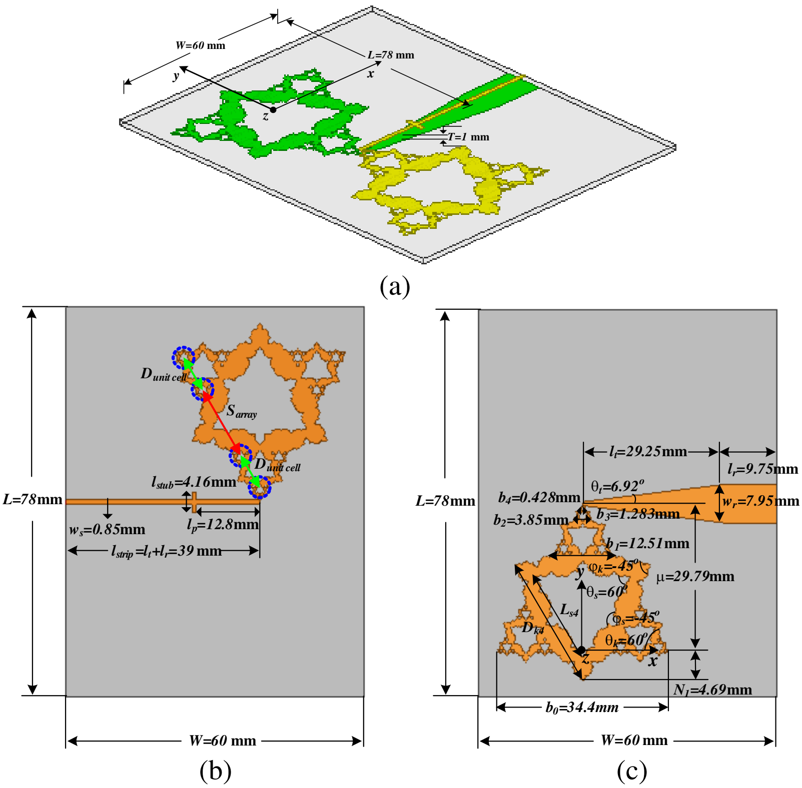
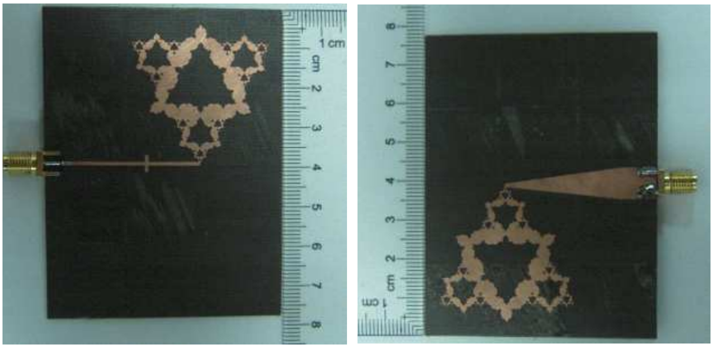



**Figure 3.** Geometry of $K_4S_4$ SKLB multifractal dipole (unit: mm). (The parts encircled by blue dotted line are sub self-similar copies of the overall geometry, which comprise unit cells of the array at $f_5$ = 9.335 GHz, with spacing $D_{unit\_cell}$ and sub-array separation $S_{array}$). (a) Southwest sideview. (yellow-top; green-bottom). (b) Top view. (c) Bottom view.

$\theta_s = 60°$, $\varphi_k = \varphi_s = -45°$; $b_0 = 34.4$ mm, $b_1 = 12.51$ mm, $b_2 = 3.849$ mm, $b_3 = 1.283$ mm, $b_4 = 0.4277$ mm; because of existence of the joint segment between the vertex and feeding Balun, height of the bow-tie $\mu = 29.79\,\text{mm} + 0.25\,\text{mm} = 30.04\,\text{mm}$; $\delta_1 = \delta_2 = 1$, $\delta_3 = 1.25$, $\delta_4 = 0.75$, $\rho_{sj} = 0.7$. The two arms of the multifractal dipole with a lineally tapered microstrip Balun [16–18] are etched on top and bottom of a Taconic TLX-5A dielectric substrate with size of 78 mm × 60 mm × 1.0 mm ($L \times W \times T$, with 35 µm copper cladding), $\varepsilon_r = 2.17 \pm 0.02$, and $\tan\delta = 0.0009$ separately, as shown in Fig. 3(a). The balanced ends are connected into the two arms while the unbalanced end is jointed with 50 Ω SMA connector. The signal trace of the Balun has width $w_s = 0.85$ mm and an orthogonal stub with length $l_{stub} = 4.16$ mm at $l_p = 12.8$ mm away from the feeding joints. The rectilinear segment of the ground of the Balun has length $l_r$ =9.75 mm and width $w_r = 7.95$ mm. The linearly tapered segment has length $l_t = 29.25$ mm, width $w_t = 7.95$ mm–0.85 mm, and pyramidal angle $\theta_t = 6.92°$, as shown in Figs. 3(b) and 3(c). The overall length and width of the SKLB multifractal dipole is $L = 68.96$ mm, $W = 56.2$ mm. The multifractal dipole prototype, as shown in Fig. 4, is fabricated by photolightprocess with a photolaser, which emits laser beam with facular diameter of 25 µm.

As shown in Fig. 5, simulated input impedance $Z_{in}$ of the $K_4S_4$ SKLB multifractal dipole displays very consistent values approximating to 50 Ω at several frequencies within band 1.5 GHz–14.5 GHz, which means remarkable multiband property (red solid-$R_{in}$, red dash-$X_{in}$). Accordingly, the simulated reflection coefficient $S_{11}$ is



(a) (b)

**Figure 4.** Prototype of $K_4S_4$ SKLB multifractal dipole (unit: mm). (a) Top view. (b) Bottom view.

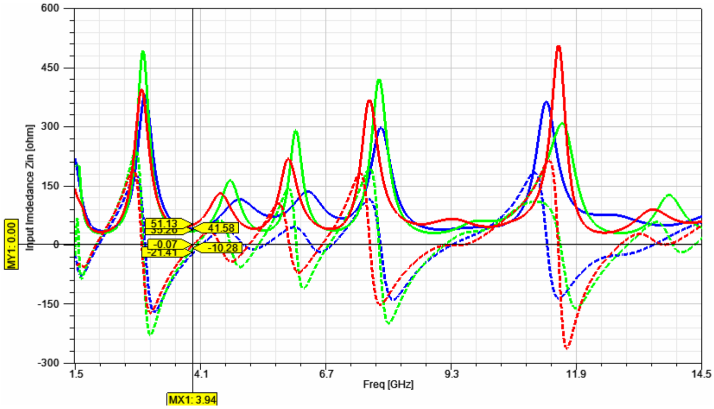



**Figure 5.** Input impedance $Z_{in}$ of $K_iS_j$ SKLB multifractal dipole (red — $K_4S_4$, green — $K_0S_4$, blue — $K_4S_0$; black-solid — $R_{in}$; thin dash — $X_{in}$).

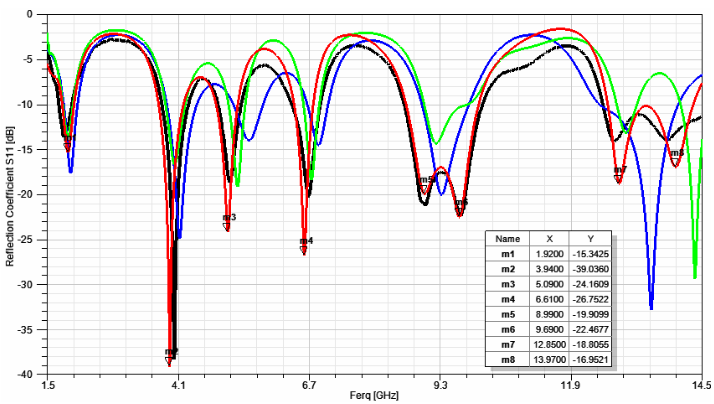



**Figure 6.** Reflection coefficient $S_{11}$ of $K_iS_j$ SKLB multifractal dipole (red — $K_4S_4$ simulated, green — $K_0S_4$, blue — $K_4S_0$, black — $K_4S_4$ measured).

shown in Fig. 6 (red solid). It seems that the dipole has eight true resonant frequencies in respect of $S_{11} \leq -15\,\text{dB}$, which corresponds to six bands with $S_{11} \leq -10\,\text{dB}$, of which $f_1$ is fundamental band, $f_2$–$f_5$ are inductive bands during iterative procedure, and $f_6$ is highest band corresponding to apex intact bow-tie, so the total matched bands: $N = 1 + (i = j = 4) + 1 = 6$ [19]. Then we measured the $S_{11}$ with Agilent E8361C vector network analyzer within the same band, also as shown in Fig. 6 (black solid). Comparably, the measured (black solid) and simulated (red solid) results of reflection coefficient $S_{11}$ are quite accordant with each other though the former shows higher values and slight shifting at some resonant frequencies. This could be mainly imputed to large ohmic loss of the Balun and copper cladding in high frequency, substrate dielectric permittivity $\varepsilon_r$ declination, fabrication tolerance and inherent error of the measurement systems.

### 3.2. Advantages over the Monofractal Counterparts

In order to ulteriorly reveal the proposed multifractal antenna's superiority over monofractal one in performance, we choose its component fractals $K_0S_4$ Sierpinski Gasket ($\rho_{Sj} = 1$, $\delta_i = 0$) and $K_4S_0$ Koch-like sided bow-tie ($\rho_{Sj} = 0$, $\varphi_k = 0°$) as its comparative counterparts because the two fractal dipoles have most similar electrical properties with it. We model the monofractal dipoles identically with the proposed $K_4S_4$ SKLB multifractal dipole, and simulate them with the same software analysis setups. The simulated and measured results of $K_4S_4$ SKLB dipole and simulated results of $K_0S_4$ and $K_4S_0$ dipoles are merged into corresponding plots for discrepancy comparison and redundancy avoidance, as shown in Figs. 5–15. Thereinto, red, black, green and blue represents simulated and measured $K_4S_4$, simulated $K_0S_4$ and $K_4S_0$ respectively.

As Fig. 5 and Fig. 6 shown, simulated input impedances $Z_{in}$ and reflection coefficients $S_{11}$ of $K_4S_4$ SKLB dipole (red) alike with that of its monofractal counterparts $K_0S_4$ Sierpinski Gasket (green) and $K_4S_0$ Koch-like sided bow-tie (blue). Especially, $K_4S_4$ resembles the minor component fractal $K_0S_4$ so much that we are susceptible to doubt the necessity of multifractal. However, the $K_4S_4$ SKLB multifactal dipole presents lower resonant frequencies, more impedance uniformity and $50\,\Omega$ proximity, which suggests further size reduction and more ideal multiband $S_{11}$. In addition, all $K_iS_j$ SKLB multifactal dipole manifest conspicuous widebands in high frequency owing to the microstrip Balun. Then we measured the radiation patterns of all the matched bands in a 3D anechoic chamber. The simulated and measured results are merged into corresponding charts for discrepancy comparison and redundancy avoidance, as illustrated in Fig. 7–Fig. 12. In these plots, bold solid, thin dash represents $Phi = 0°$, $Phi = 90°$ principle cut-plane respectively and red, black, green, blue denotes simulated, measured $K_4S_4$, $K_0S_4$ and $K_4S_0$ in sequence.

As shown in Figs. 7–12, measured (black) and simulated (red) results of the gain patterns are also very accordant with each other, but the former have a little asymmetry, which should be attributed to fabrication errors and imperfect test conditions. The maximum

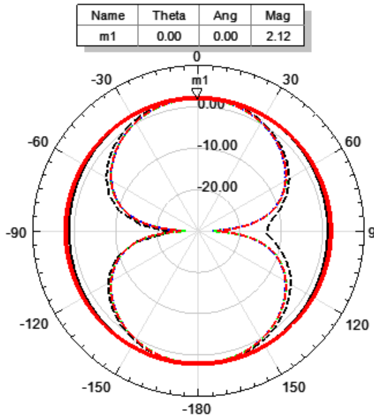



**Figure 7.** Gain patterns of $K_iS_j$ at $f_1$ (red — $f_1 = 1.92$ GHz-simulated $K_4S_4$, black — $f_1 = 1.858$ GHz-measured $K_4S_4$, green — $f_1 = 1.92$ GHz-$K_0S_4$, blue — $f_1 = 1.97$ GHz-$K_4S_0$; solid — $Phi = 0°$-$XOZ$, dash — $Phi = 90°$-$YOZ$).

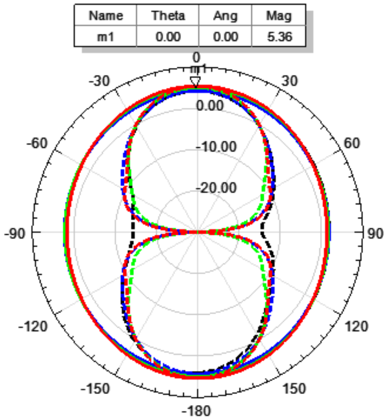



**Figure 8.** Gain patterns of $K_iS_j$ at $f_2$ (red — $f_2 = 3.94$ GHz-simulated $K_0S_4$, black — $f_2 = 4.027$ GHz-measured $K_4S_4$, green — $f_2 = 4.04$GHz-$K_0S_4$, blue — $f_2 = 4.13$ GHz-$K_4S_0$; solid — $Phi = 0°$-$XOZ$, dash — $Phi = 90°$-$YOZ$).

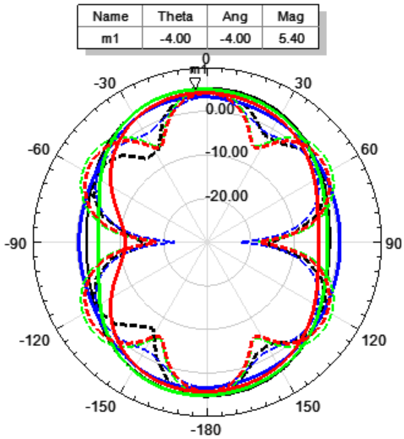



**Figure 9.** Gain patterns of $K_iS_j$ at $f_3$ (red — $f_3 = 5.09$ GHz-simulated $K_4S_4$, black — $f_3 = 5.124$ GHz-measured $K_4S_4$, green — $f_3 = 5.28$ GHz-$K_0S_4$, blue — $f_3 = 5.52$ GHz-$K_4S_0$; solid — $Phi = 0°$-$XOZ$, dash — $Phi = 90°$-$YOZ$).

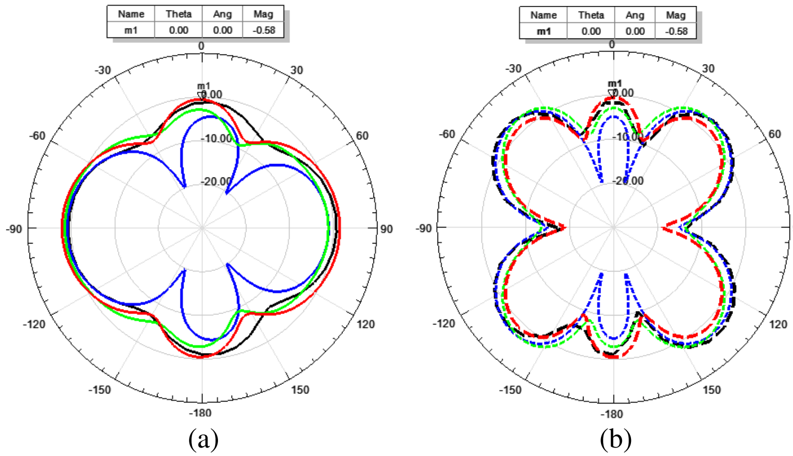



**Figure 10.** Gain patterns of $K_iS_j$ at $f_4$ (red — $f_4$ = 6.61 GHz-simulated $K_4S_4$, black — $f_4$ = 6.684 GHz-measured $K_4S_4$, green — $f_4$ = 6.75 GHz-$K_0S_4$, blue — $f_3$ = 6.89 GHz-$K_4S_0$). (a) $Phi = 0°$-$XOZ$. (b) $Phi = 90°$-$YOZ$.

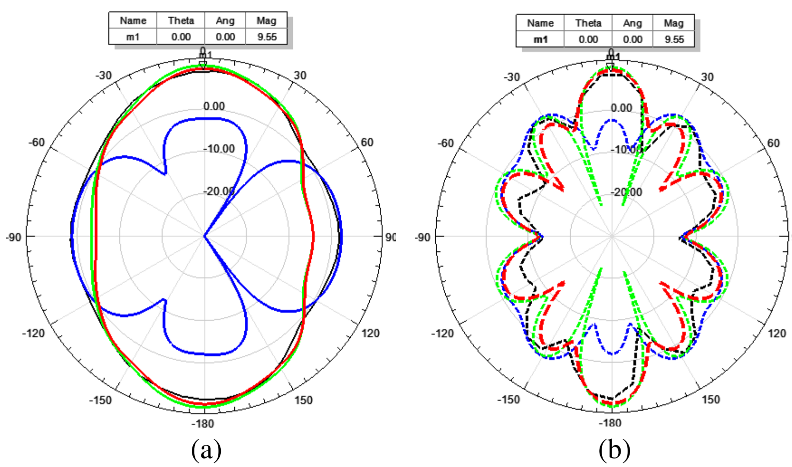
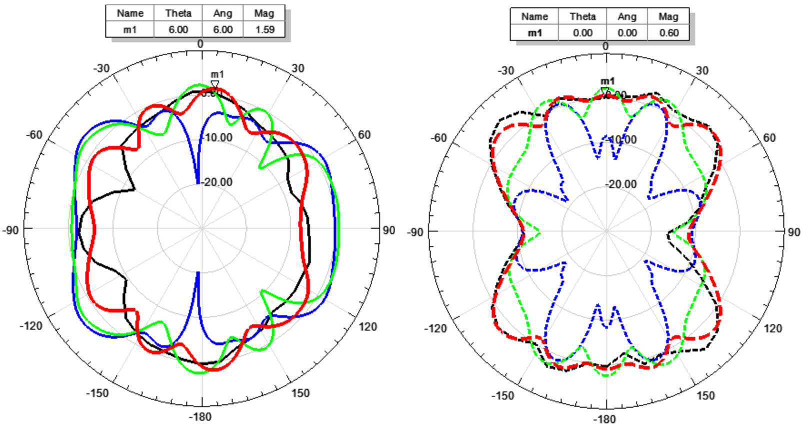



**Figure 11.** Gain patterns of $K_iS_j$ at $f_5$ (red — $f_5$ = 9.335,GHz-simulated $K_4S_4$, black — $f_5$ = 9.333,GHz-measured $K_4S_4$, green — $f_5$ = 9.23,GHz-$K_0S_4$, blue — $f_5$ = 9.34 GHz-$K_4S_0$). (a) $Phi = 0°$-$XOZ$. (b) $Phi = 90°$-$YOZ$.




(a) (b)

**Figure 12.** Gain patterns of $K_iS_j$ at $f_6$ (red — $f_6$ = 13.41 GHz-simulated $K_4S_4$, black — $f_6$ = 13.24 GHz-measured $K_4S_4$, green — $f_6$ = 13.67 GHz-$K_0S_4$, blue — $f_6$ = 13.48 GHz-$K_4S_0$). (a) $Phi = 0°$-$XOZ$. (b) $Phi = 90°$-$YOZ$.

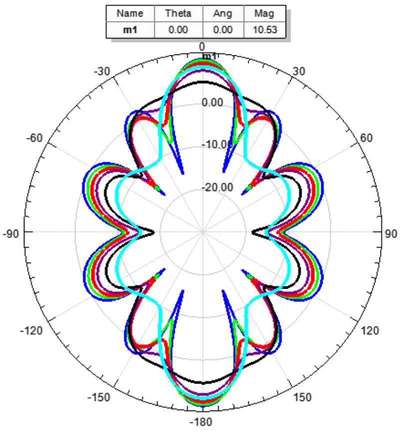



**Figure 13.** $E$-plane ($Phi = 90°$-$YOZ$) gain patterns of $K_4S_4$ SKLB multifractal dipole at $f_5$ = 9.335 GHz with sweep of $\delta_3$ ($G_{\max}$ = 10.53 dBi; blue — $\delta_4 = 0.6$, green — $\delta_4 = 0.8$, red — $\delta_4 = 1.0$, purple — $\delta_4 = 1.2$, black — $\delta_4 = 1.4$, cyan — $\delta_4 = 1.6$)

radiations are in the vicinity of the normal direction ($+Z$). In addition, similarity and asymmetry are also observed from the simulated patterns. Gain patterns of $K_4S_4$ SKLB dipole at $f_1$ and $f_2$ are omnidirectional in $XOZ$ ($Phi = 0°$, $H$-plane) and doughnut-shaped in $YOZ$ ($Phi = 90°$, $E$-plane), as shown in Figs. 7, 8. Gain patterns of $K_4S_4$ SKLB dipole at $f_3$ and $f_4$ are quasi-omnidirectional in $XOZ$ ($Phi = 0°$, $H$-plane), cloven doughnut-shaped or dented doughnut-shaped in $YOZ$ ($Phi = 90°$, $E$-plane), as shown in Figs. 9, 10. Gain patterns of $K_4S_4$ SKLB dipole at $f_5$ are elliptoid (with long axis in $Z$-axis) in $XOZ$ ($Phi$ =0°, $H$-plane), cloven doughnut-shaped in $YOZ$ ($Phi$ =90°, $E$-plane), as shown in Fig. 11. Gain of $f_5$, which is as high as 9.55 dBi, indicates that there is a broadside array formed by self-similar elements with smaller scale in this multifractal geometry, as depicted in Fig. 3(b). For validity of this viewpoint, parametric sweep is undertook for $\delta_4$, which is the most relevant parameter to $f_5$, and side lobe level of the gain pattern in $E$-plane decreases with $\delta_4$ distinctly, which indicates that the pattern closely correlates to element spacing, excitation amplitude, and phase discrepancy just like its conventional counterpart half-wavelength broadside dipole array (4 unit cells spaced about 0.8–0.9$\lambda$), as illustrated in Fig. 13. For optimal synthesis of gain pattern, the four lateral unit cells should be equidistant, namely $D_{unit\_cell} = S_{array}$, so from formula (7), we acquire: $\theta_k = 60°$, $\delta_3 = 1$, $\delta_4 = 2/3$. From Fig. 13, we can see that $\delta_4 = 0.6$ is very approximate to this optimum value. Such array with element coalition and inequable excitation should be a very peculiar phenomenon emerging in multifractal antenna.

Gain patterns of $K_4S_4$ at $f_6$ are also elliptoid (with long axis in $Z$-axis) in $XOZ$ ($Phi = 0°$, $H$-plane), rippled doughnut-shaped in $YOZ$ ($Phi = 90°$, $E$-plane), as shown in Fig. 12.

Such radiation patterns above suggest that $K_4S_4$ SKLB dipole operates as a half-wavelength dipole at all matched bands except $f_4(\approx 1.5 \times \lambda)$. It doesn't present identical gain patterns in multiband as the standard half-wavelength dipole due to fringe or end effect [20] of the smaller scale parts corresponding to the whole fractal similar geometry. Slight asymmetry of gain patterns could be attributed to the antipodal configuration, the feedline's $+X$ extension and noideal $\pm 180°$ phase difference at balanced ends of the Balun.

Gain patterns of $K_4S_4$ SKLB dipole very resembles that of $K_0S_4$ and $K_4S_0$ in low frequency, as shown in Figs. 7–9, but it displays a little improved gains than $K_0S_4$ and enhances distinctly than $K_4S_0$, which maximizes in $-X$ direction in $H$-plane and nulls in $Z$ direction in high frequency, as shown in Fig. 10–Fig. 12. Analogical gain patterns of multibands indicate SKLB dipole's better identity in radiation than monofractal counterparts like [11] and [12]. Likewise, remarkable enhancements like higher gain and better directivity over its component monofractals $K_4S_0$ and $K_0S_4$ are acquired for these multiband patterns. The $K_4S_4$ SKLB dipole behaves more like $K_0S_4$

Sierpinski Gasket within the overall band though this monofractal is arranged in inferior order. The fact indicates that it's the panoramic monofractal rather than the local monofractal dominates the mutlifractal's properties.

For convenient acquisition of thorough properties, the simulated resonant results are tabulated in Table 1.

As shown in Table 1, $f_1$, $f_2$ and $f_3$ just fall upon very useful bands with considerable relative bandwidth such as UMTS, WiFi, and WiMAX, which means $K_4S_4$ SKLB dipole will be a competitive multiband antenna candidate at least a triband antenna with better

**Table 1.** Simulated resonant properties of $K_iS_j$ SKLB multifractal dipole.

| | | | | |
|---|---|---|---|---|
| $K_4S_4$ | $f_i$ (GHz) | $f_1$ | $f_2$ | $f_3$ |
| | | 1.92 | 3.94 | 5.09 |
| | $f_{n+1}/f_n$ | - | **2.052** | **1.292** |
| | $R_{in}$ (Ω) | 35.76 | 51.13 | 44.52 |
| | $S_{11}$ (dB) | −15.34 | −39.04 | −24.16 |
| | BW (MHz) | 242, 12.61% | 510, 12.95% | 460, 9.04% |
| | Gain (dBi) | 2.12 | 5.36 | 5.40 |
| $K_0S_4$ | $f_i$ (GHz) | $f_1$ | $f_2$ | $f_3$ |
| | | 1.92 | 4.04 | 5.28 |
| | $f_{n+1}/f_n$ | - | **2.104** | **1.307** |
| | $R_{in}$ (Ω) | 33.21 | 37.80 | 40.62 |
| | $S_{11}$ (dB) | −13.69 | −17.93 | −19.16 |
| | BW (MHz) | 200, 10.42% | 391, 9.68% | 336, 6.36% |
| | Gain (dBi) | 2.04 | 5.04 | 5.06 |
| $K_4S_0$ | $f_i$ (GHz) | $f_1$ | $f_2$ | $f_3$ |
| | | 1.97 | 4.13 | 5.52 |
| | $f_{n+1}/f_n$ | - | **2.096** | **1.337** |
| | $R_{in}$ (Ω) | 41.53 | 44.77 | 73.69 |
| | $S_{11}$ (dB) | −17.60 | −24.88 | −14.0 |
| | BW (MHz) | 275, 13.96% | 600, 14.53% | 600, 10.87% |
| | Gain (dBi) | 2.08 | 4.28 | 3.36 |

| | | | | |
|---|---|---|---|---|
| $K_4S_4$ | $f_i$ (GHz) | $f_4$ | $f_5$ | $f_6$ |
| | | 6.61 | 9.335 (8.99–9.68) | 13.41 (12.85–13.97) |
| | $f_{n+1}/f_n$ | **1.299** | **1.412** | **1.437** |
| | $R_{in}$ ($\Omega$) | 45.68 | 59.29 | 41.50 |
| | $S_{11}$(dB) | −26.75 | −19.91 | −18.81 |
| | BW (MHz) | 400, 6.05% | 1460, 15.62% | 1730, 12.85% |
| | Gain (dBi) | 1.50 | 9.55 | 1.59 |
| $K_0S_4$ | $f_i$ (GHz) | $f_4$ | $f_5$ | $f_6$ |
| | | 6.75 | 9.23 | 13.67 (12.99–14.35) |
| | $f_{n+1}/f_n$ | **1.278** | **1.367** | **1.481** |
| | $R_{in}$ ($\Omega$) | 39.29 | 35.16 | 32.53 |
| | $S_{11}$ (dB) | −18.37 | −14.32 | −13.17 |
| | BW(MHz) | 318, 4.71% | 868, 9.41% | 1013, 7.41% |
| | Gain (dBi) | −2.94 | 10.28 | 2.26 |
| $K_4S_0$ | $f_i$ (GHz) | $f_4$ | $f_5$ | $f_6$ |
| | | 6.89 | 9.34 | 13.48 |
| | $f_{n+1}/f_n$ | **1.248** | **1.356** | **1.443** |
| | $R_{in}$ ($\Omega$) | 71.54 | 41.19 | 52.30 |
| | $S_{11}$(dB) | −14.50 | −19.97 | −32.80 |
| | BW (MHz) | 416, 6.04% | 933, 9.99% | 1569, 11.64% |
| | Gain (dBi) | −4.87 | −2.27 | −8.54 |

gain patterns and higher gains than that of [21–25] at $f_1$, $f_2$, and $f_3$ respectively, for many wireless communication applications; larger percentage bandwidths than $K_0S_4$ and $K_4S_0$ at $f_5$ and $f_6$ validate band widening property of the proposed multifractal; resonant impedances $R_{in}$ are all very approximate to $50\,\Omega$ and reflection coefficients $S_{11}$ are all less than $-15$ dB, which suggest conspicuous multifractal impedance uniformity; adjacent frequency ratio presents remarkable multi-values and consistency, which denotes distinct multifractal multiband characteristics.

At the end, the radiation efficiency of $f_1$, $f_2$ and $f_3$ are measured and tabulated in Table 2.

**Table 2.** Measured gain and efficiency of $f_1$, $f_2$ and $f_3$ ($E$, $H$-plane).

| $f$ (GHz) | Gain ($E$, dBi) | HPBW ($E$, °) | Gain ($H$, dBi) | HPBW ($H$, °) | Effic ($\eta$) |
|---|---|---|---|---|---|
| 1.88 | 2.04 | 79.0 | 2.04 | - | 96% |
| 1.92 | 2.10 | 78.5 | 2.10 | - | 97% |
| 1.96 | 2.07 | 78.5 | 2.07 | - | 95% |
| 3.90 | 4.57 | 32 | 4.57 | - | 92% |
| 3.94 | 4.50 | 32 | 4.50 | - | 92% |
| 3.98 | 4.55 | 33 | 4.55 | - | 92% |
| 5.05 | 5.32 | 21.6 | 5.32 | 60.5 | 86% |
| 5.09 | 5.36 | 21 | 5.36 | 61 | 87% |
| 5.14 | 5.43 | 22 | 5.43 | 60 | 87% |

As Table 2 shown, the radiation efficiency $\eta$ at the test bands is high and decreases with $f$, because loss increases with frequency. In conclusion, the $K_4S_4$ SKLB multifractal dipole does not degrade with iteration growth and frequency increase in performance like bandwidth, gain and efficiency.

## 4. CONCLUSION

Koch-like curve and Sierpinski Gasket are coalesced into a fire-new Sierpinskized Koch-like sided bow-tie multifractal (SKLB) in main-minor way with correlative and variable fractal scale ratios [26]. A $K_4S_4$ SKLB multifractal dipole with variable fractal sale ratios fed by a linearly tapered microstrip Balun, etched on a Taconic TLX-5A dielectric substrate with dimension of 78 mm (68.96 mm) × 60 mm (56.2 mm) × 1.0 mm ($L \times W \times T$, with 35 µm copper cladding), $\varepsilon_r = 2.17 \pm 0.02$, and $\tan\delta = 0.0009$, is designed, simulated, fabricated and measured. If a higher permittivity substrate, such as FR4, is chosen, more compact configuration will be obtained [27], so that it will be suitable for small multiband wireless device such as RFID [28] and USB dongle [29]. The well accordant results from measurement and experiment corroborate validity of the design with Ansoft HFSS$^{\text{TM}}$ v.13 and the multifractal antenna's superiority and advantages over its monofractal counterparts in impedance uniformity, bandwidth broadening, directivity amelioration, dimension shrinkage, and efficiency enhancement. Six well matched bands ($S_{11} \leq -10$ dB) with moderate gain (2.12 dBi–9.55 dBi) and high efficiency (87%–97%)

are obtained within band 1.5 GHz–14.5 GHz, and all the bands $f_1 =$ 1.92 GHz (1.8 GHz–2.042 GHz; 242 MHz, 12.61%, PCS1900 + UMTS), $f_2$ = 3.94 GHz (3.71 GHz–4.22 GHz; 510 MHz, 12.95%, WiMAX), $f_3$ = 5.09 GHz (4.84 GHz–5.3 GHz; 460 MHz, 9.04%, WiMAX), $f_4$ = 6.61 GHz (6.4 GHz–6.8 GHz; 400 MHz, 6.05%), $f_5$ = 9.335 GHz (8.62 GHz–10.08 GHz; 1460 MHz 15.62%), $f_6$ = 13.41 GHz (12.6 GHz–14.33 GHz; 1730 MHz, 12.85%), in which peculiar array effect is observed, are generally useful. The multibands are omnidirectional or quasi-omnidirectional in $H$-plane ($Phi = 0°$, $XOZ$) and doughnut-shaped or dented doughnut-shaped in $E$-plane ($Phi = 90°$, $YOZ$). Moreover, this multifractal antenna possesses compactness, simpleness, and lightweight. So it is an attractive candidate for applications such as PCS, IMT2000, UMTS, WLAN, WiFi, WiMAX and other fixed or mobile wireless multiband communication systems [30].

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